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Government Size and Economic Growth in an Endogenous Growth Model with Rent-seeking*

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Abstract

We explore the relationship between government size and economic growth in an endogenous growth model with human capital and an unproductive capital which facilitates rent-seeking. With exogenous as well as endogenous time discounting, we find a non-monotonic relationship between the size of government and economic growth. We find that with very high (low) discounting, there is a unique low (high) growth equilibrium, regardless of the size of government. For the intermediate range of discounting, there are multiple equilibria and the growth outcome depends on the size of government. With endogenous time discounting, the growth outcome is path-dependent and depends on the level of inherited human capital. However, there is only one stable growth regime and the economy endogenously switches to it. When the institutional constraints on rent seeking are not extremely high, the stable regime is the one in which there is a high-growth equilibrium for a smaller size of the government and for larger size, both the high-growth and the low-growth equilibrium coexist. When the institutional constraints on rent seeking are extremely high, there exists only a unique high-growth equilibrium irrespective of the size of government. Furthermore, economies with bigger size of the government and/or with poor quality institutions will take longer to endogenously switch to this stable growth regime.

Keywords: Government size; Rent-seeking; Economic Growth; Human capital; Discounting.

JEL codes: O41, H11, D72, D90, J24, O43.

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1 Introduction

The relationship between government size and economic growth has traditionally been a well debated topic in economics. Empirical evidence is mixed as a higher government size is associated with accelerated economic growth in some cases and retarded growth in the others (see Nijkamp and Poot, 2004; Colombier, 2009; and Facchini and Melki, 2013 for a comprehensive meta-analysis of studies). This, in fact, supports both the Pigovian view of a benevolent government and the Public choice view of a distortive government. In the wake of Grossman (1988), this literature has focused more on the non-linearity of the relationship between the two, where the positive effect is dominant until the government size does not exceed some threshold level. Theoretically, the notion of an optimal size of government was popularized by Barro (1990) and Armey (1995) who illustrated an inverted U-shaped relationship between government size and economic growth. By its implication, the relationship between the size of government and economic growth is non-monotonic. Building on the rent-seeking literature (Krueger, 1974; Ehrlich and Lui, 1999; Wadho, 2014), we develop an endogenous growth model in which the size of government affects the accumulation of both, the productive and the unproductive capital, which in turn affects the steady-state (equilibrium) as well as the rate of economic growth. The idea is that government intervention creates prospects for resource transfer due to which individuals may have an incentive to invest in the unproductive capital. Individuals may invest in the unproductive capital to extract these resources and/or to resist the transfer of their resources. In this paper, we model individuals as facing a trade-off between investing in human capital, which generates higher returns in labor market, and an unproductive capital, which helps in extracting rents created due to the government intervention.

Literature on corruption and economic growth suggests that corruption is detrimental for growth (Mauro, 1995; Goel and Nelson, 1998; Ehrlich and Lui, 1999; Wadho, 2016). The Public choice view of government intervention posits government intervention in the market as a third common factor simultaneously affecting both corruption and economic growth. Government intervention creates opportunities for private individuals as well as for public officials to seek private benefits at the expense of social welfare. These prospects of rent-seeking promote investment in unproductive activities which are primarily meant to extract rents created due to the government intervention. In such situations, individuals face a trade-off between investing in a productive capital, such as human capital, which enhances their set of skills and increases their earnings, and an unproductive capital, which facilitates rent-seeking. In Ehrlich and Lui (1999) and Wadho (2014), investment in an unproductive capital negatively affects the accumulation of human capital and economic growth.[1] This creates a distinction between the two forms of capital where one accelerates economic growth and the other retards it.

Human and unproductive capital are also believed to differ in another important aspect — the timeline of investment and its returns. Human capital is considered as a future oriented capital and is generally associated with a delayed realization of returns. Unproductive capital for rent-seeking, on the other hand, is associated with a relatively smaller time lag between investment and returns.[2] Hence, the returns to investment in these two types of capital depend not only on market factors and the size of government, but also on individuals' time preferences. Recent research on the underinvestment in productive capitals directs towards such behavioral issues which limit investment decisions by changing the decision making

[1] Both Ehrlich and Lui (1999) and Wadho (2014) label this unproductive capital as 'political capital'.

[2] This, for example, can be taken as having links with public officials who can help in tax avoidance/evasion which may not need a number of years of accumulation.

process (Mullainathan, 2005; Duflo, 2006). Becker and Mulligan (1997) suggest that the interaction between time preferences and individual characteristics, such as education, wealth, and addictions, can partially explain this observation. They argue that a lower rate of time preference enables individuals to discount distant utilities less; making investment in future oriented capitals more attractive.

Empirical literature on individuals' time preferences proposes some possible correlates of time discounting. Brown and van der Pol (2015) find a positive correlation between the intergenerational (i.e. parents' and their children's) rate of time preference using data from an Australian household survey. The same was observed by Webley and Nyhus (2006) for a Dutch panel dataset. Harrison et al. (2002) provide evidence for the presence of a large difference between the discount rates of skilled and unskilled individuals, with those who have skills having a significantly lower discount rate (lower time discounting). The correlation between education and lower discounting of future is also highlighted by Bauer and Chytilova (2010), who use the data from Ugandan villages, and by Kirby et al. (2002), using data on Tsimane Amerindians of the Bolivian rainforest. Similarly, Perez-Arce (2011), using data on individuals seeking admission in public colleges in Mexico, shows that successful applicants were, on average, more patient than those who were not admitted. Kirby et al. (2002) find that an individual's discount rate decreases with parental (father's) education. Bjorklund and Salvanes (2011, ch:3) indicate that an individual's incentive to accumulate human capital is influenced by the stock of her parental human capital. On individuals' risk aversion, Hryshko et al. (2011) show that individuals' risk aversion is affected, in part, by parental education. This literature further highlights the presence of threshold effects. Hryshko et al. (2011) observe that parental education beyond grade 11 has a significant impact on an individual's risk aversion. Haveman et al. (1991) and Manski et al. (1992) show that parental completion of high school and one or two years of post-secondary schooling are typically found to have a larger effect on children's schooling when compared to other levels of parental education. Becker and Mulligan (1997) is amongst the most notable papers in economic theory which model the correlation between education and discount rate. In growth models, endogenous discounting is modeled as a function of physical capital (Haaparanta and Puhakka, 2004), level of consumption (Sarkar, 2007), a generic theoretical construct of 'future-oriented' capital (Stern, 2005), average propensity to consume (Zee, 1997), and individuals' own human capital (Bauer and Chytilova, 2008).

In this study, we model individuals' time discounting as endogenous in the light of the three premises highlighted in the recent empirical literature; a positive correlation between education and low discounting, a positive correlation between children's and parents' discounting, and a positive correlation between parental education and low discounting. Specifically, we assume that parents affect children's time discounting. We do not explicitly model the channels through which parents affect children's time discounting, but rather rely on a broader notion that individuals' future orientation is associated with their parents' level of education.

Our model's settings have some features in common with those of Ehrlich and Lui (1999) and Wadho (2014), where the former looks at the link between the accumulation of human capital, political (un productive) capital, and economic growth in the presence of government intervention, and the latter looks at the link between the accumulation of human capital, political capital, and economic growth in a setting in which natural resources are the source of rents. Our model's settings, however, differ from their's in several ways. First, as suggested by the rent-seeking literature, government intervention in the economy creates prospects for transfers and reallocation. Individuals attempt to do both – resist a transfer away from them and extract from the pool of transfers. Second, we consider individuals' time discounting as endogenous, shaped by the level of parental education. Third, in Ehrlich and Lui (1999), there is no public good provision by the government which implies that a higher government size always increases individuals' incentive to seek rents by appropriating resources, whereas we model a redistributive government with imperfect administrative controls and institutional checks which creates a trade-off for rent-seeking.

We also consider rent-seeking as illegal and rent-seekers run the risk of getting caught and punished. Thus, the quality of institutions, which determines the probability of getting caught and being punished, will also affect individuals' incentive to invest in rent-seeking and human capital. We model institutional quality as a set of ex-anté administrative controls, which constrain individuals from resisting (evading) the government intervention, and ex-post controls, which help in the detection and punishment of rent-seekers. Similar to the concept of institutions in Acemoglu et al. (2003), quality of institutions here implies how strong or weak are the constraints on rent-seeking.

In the first part of the paper, we consider exogenous time discounting and show that the relationship between the size of government and economic growth is non-monotonic, and growth outcomes, crucially, depend on individuals' time discounting. However, this relationship is different from the traditional U-shaped relationship discussed in the literature. We find that for very high and very low levels of time discounting, there is a unique high-growth and a unique low-growth equilibrium, respectively. In both cases, the equilibrium is unique and stable and not dependent on the size of government. However in the case of low-growth equilibrium, the growth rate decreases with the size of government. For the intermediate levels of individuals' time discounting, there are multiple equilibria with three stable growth regimes that are demarcated by the level of individuals' time discounting. In each of the three growth regimes, the equilibrium (high-growth, low-growth, or both) depends on the size of government. In all of the regimes, a bigger size of the government results in the low-growth equilibrium.

With endogenous time discounting, we show that growth outcomes are path-dependent and depend on the level of inherited human capital. For very high and very low levels of the inherited human capital, there is a unique high-growth and a unique low-growth equilibrium, respectively. In both cases, the equilibrium does not depend on the size of government. For intermediate levels of the inherited human capital, there are three different growth regimes demarcated by the levels of inherited human capital. In all three regimes, the equilibrium depends on the size of government in which a bigger size of the government will result in the low-growth equilibrium.

The path-dependent growth outcome with threshold effects is in line with the seminal work of Azariadis and Drazen (1990). However, our results differ from theirs in a number of important aspects. First, rather than affecting the productivity of the human capital accumulation technology, the inherited human capital affects individuals' patience which makes investment in the productive human capital with delayed returns more valuable. This is in line with the empirical findings of Haveman et al. (1991) and Manski et al. (1992) on the impact of parental education on children's schooling and with the results of Dohmen et al. (2010) and Hryshko et al. (2011) on individuals' risk aversion. Second, in our results, the thresholds of human capital which demarcate different growth regimes are endogenous and depend on the quality of institutions. This suggests that economies with better quality institutions will end up in the high-growth equilibrium regime with relatively smaller levels of the inherited human capital.

Finally, the main difference between the results with exogenous time discounting and those with endogenous discounting is that with endogenous discounting, there is only one stable growth regime and the economy endogenously switches to this growth regime. Which of the equilibrium regimes is stable, crucially, depends on the level of ex-anté institutional constraints and there are two possibilities. First, when the ex-anté institutional constraints are not extremely high, the stable growth regime is the one in which both the high and the low growth equilibrium exist and are stable. The high-growth equilibrium exists for a relatively small government size, and for a relatively large size of the government, both high-growth and low-growth equilibrium co-exist. On the other hand, when the ex-anté institutional constraints are extremely high, the stable growth regime is the one in which only a unique high-growth equilibrium exists. Furthermore, the time an economy takes to switch from one growth regime to another depends on the size of government as well as on the quality of institutions. Economies with a bigger government size and/or with poor quality institutions will take longer to endogenously switch to another growth regime.

The rest of the paper proceeds as follows. In the next section, we define our model. Section 3 solves the model and characterizes the different equilibria with exogenous time discounting, whereas section 4 characterizes the different equilibria with endogenous discounting. The last section concludes the paper.

2 Description of the Economy

2.1 Households

There is an overlapping generations economy of two period lived individuals with a constant population of size n. All individuals are identical within each generation and are endowed with one divisible unit of time in each period. When young, they divide their time between schooling, accumulating unproductive capital and working; and when old they spend their entire time working.

All individuals born at $t = 1, 2, ...$ have identical preferences represented by the utility function

$$U_t = \ln(c_{1t}) + \beta \ln(c_{2t}) \tag{1}$$

where c_{1t} and c_{2t} is the consumption in first and second period, respectively. $\widetilde{\beta} \leq \beta \leq 1$ is the discount factor with $\widetilde{\beta} > 0$ its minimum level.[3]

2.2 Technologies

Human capital is the engine of growth. The modeling of human capital is similar to that in the standard models in economic growth (i.e. Lucas, 1988; Ehrlich and Lui, 1999; Wadho, 2014) in which it is associated with a delayed realization of returns. Individuals, when young, accumulate human capital and get returns only in the second time period. Human capital is generated by

$$H_{i2t} = AH_{i1t}h_{it} \tag{2}$$

[3] In Section 4 we model the discount factor as endogenous. A discount factor which is strictly positive and which does not exceed one is a convention in economic models involving inter-temporal decision-making (see Lucas, 1988; Galor and Zeira, 1993; Redding, 1996; Eicher et al., 2009 for examples).

where H_{i1t} denotes the inherited human capital, h_{it} is the fraction of time invested in creating future human capital, and $A > 1$ is a technological parameter.

Symmetrically, the unproductive capital is generated by

$$Q_{i2t} = Q_{i1t} = BQ_{i0t}q_{it} \tag{3}$$

where Q_{i0t} denotes the inherited stock of unproductive capital, q_{it} is the fraction of time invested in the accumulation of unproductive capital, $B > 1$ is a technological parameter. We assume that $Q_{i2t} = Q_{i1t}$, which implies that there is no depreciation of the unproductive capital.[4]

Both human and unproductive capital are accumulated with symmetric technologies, however, they differ in two important dimensions – their effect on production, and the timeline of their returns. Human capital augments the set of skills of individuals which increases their productivity at work. Typically, it involves a number of years of accumulation (for example, reaching the tertiary level), and the returns to it are associated with a relatively delayed realization. On the other hand, unproductive capital in our settings is similar to the concept of political capital in Ehrlich and Lui (1999) and to rent-seeking in Krueger (1974). This form of capital helps in rent-seeking and does not directly or indirectly facilitate production. In fact, as we show in the subsequent sections, it negatively affects the accumulation of human capital as well as the output. Generally, developing acquaintances[5] and building links is a relatively quick process when compared to acquiring education. Thus, unlike human capital, returns to the unproductive capital are not subject to delayed realization such that the unproductive capital is accumulated and its returns are realized in the same time period.

There is a single final good produced with two different technologies. Individuals, when young, work in the unskilled sector where the output is given by

$$Y_t^u = \sum_{i=1}^{n}(1 - h_{it} - q_{it}) \tag{4}$$

On the other hand, adults works in the skilled sector where the output is given by

$$Y_t^s = \sum_{i=1}^{n} \gamma H_{i2t} \tag{5}$$

where $\gamma > 1$ and implies that the wage (per unit of effective labor) paid in the skilled sector is greater than the wage in the unskilled sector.

Total output of the economy at time τ is thus the sum of the aggregate skilled sector output and the aggregate unskilled sector output

$$Y_\tau = Y_{\tau,t-1}^s + Y_{\tau,t}^u$$

2.3 Government Intervention and Rent-seeking

As in Ehrlich and Lui (1999), we model the size of government as all economic transactions that are subject to government intervention. There is a fraction θ of all economic activities whose proceeds are

[4] This will remain the same if we allow the unproductive capital to depreciate and assume that the new investment in it is equal to the depreciated capital, what we call as the break-even investment.

[5] Such 'friendships' are purely transactional in nature in which individuals develop contacts only for the purpose of extracting certain benefits, exhibiting opportunistic behavior.

acquired by the government, which, it then redistributes and runs a balanced budget.[6] We assume that receipts from each sector are distributed equally among the individuals working in that sector.[7] Given this, the balanced budget condition implies equating receipts, G_τ with transfers, R_τ

$$\begin{aligned} G_\tau &= R_\tau \\ \theta Y_\tau &= n\left(\frac{\theta Y^s_{\tau,t-1}}{n}\right) + n\left(\frac{\theta Y^u_{\tau,t}}{n}\right) \end{aligned}$$

The way we model the redistributive government intervention, it does not affect the equilibrium payoffs. However, as we show in the next section, it can generate a market failure where individuals invest in the costly activity of rent-seeking. The idea is that the prospects of a transfer associated with the government intervention make individuals seek rents and invest in the unproductive capital. This also implies that even when the transfer itself is costless, its prospects lead to the wasteful investment in rent-seeking. Tullock (1971:642) argues:

> "The transfer itself may be costless, but the prospect of the transfer leads individuals and groups to invest resources in either attempting to obtain a transfer or to resist a transfer away from themselves."

Our treatment of rent-seeking is along the lines of Tullock's argument, where an investment in the unproductive capital helps individuals in: i) resisting a transfer away from themselves, and ii) obtaining a transfer. Moreover, such type of rent-seeking is deemed illegal and there are constraints put in place by the law enforcement authorities to deter the resistance to transfers by rent-seekers and to penalize them when caught.

(i) Resisting a transfer away

An individual's ability to resist a transfer θ_i depends on the relative strength of her unproductive capital, the strength of administrative controls which deter such resistance represented by α, and the proportion of dishonest agents (rent-seekers) in the society, d_t.

$$\theta_{i\tau t} = \left\{ \begin{array}{ll} \theta & \text{if } q_{it} = 0 \\ \theta\left[1 - d_t\left(\frac{Q_{i\tau t}}{\overline{Q}_{\tau t}} - \alpha\right)\right] & \text{if } q_{it} > 0 \end{array} \right\} \tag{6}$$

where $\tau = 1, 2$ and $0 \leq \alpha \leq 1$, $0 \leq d_t \leq 1$, $Q_{i\tau t}$ is individual level, and $\overline{Q}_{\tau t}$ is the average level of unproductive capital.

Individuals with no investment in the unproductive capital have no ability to resist government intervention and they end up transferring resources which are equal to the level of government intervention, θ. On the other hand, those individuals who invest in the unproductive capital may end up paying less than, equal to, or even greater than the level of government intervention, θ. d_t, with a negative sign represents the positive externality stemming from rent seekers' strength in numbers — it becomes easier to evade the government intervention when many are evading. Given d_t, how much an individual pays depends on the strength of her unproductive capital relative to the extent of administrative controls, a. Individuals

[6] We abstain from the use of term 'taxation' in our discourse throughout and instead rely on much broader notion of 'government intervention'.

[7] This is to focus on a similar sort of a market failure as in the homogenous case of Ehrlich and Lui (1999) where, although optimal à priori, the investment in political capital yields zero equilibrium net returns from corruption.

with $\frac{Q_{i\tau t}}{Q_{\tau t}} > \alpha$ end up paying less than θ, whereas those with $\frac{Q_{i\tau t}}{Q_{\tau t}} < \alpha$ end up paying more than θ.[8] The strength of administrative controls, α is a form of 'ex-anté' institutional constraints, which, in the present context may refer to the competence of civil servants, their independence from being swayed by pressure groups, lobbies, and political parties.[9]

(ii) Obtaining a transfer

In obtaining a transfer, rent-seekers exert two externalities on each other. First, there is a positive externality; a higher number of corrupt individuals make it easier to divert government receipts. In particular, we assume that the fraction of rents diverted is a function of the proportion of rent-seekers, that is $P_t(d_t) = d_t[1 - \alpha(1 - d_t)]$, where $0 \leq P_t \leq 1$. With this functional form, the diverted share is an increasing function of the number of rent-seeking individuals, with $P_t(0) = 0$ and $P_t(1) = 1$. This implies that in the absence of rent-seeking, there is no leakage of the revenue and all of it is redistributed, and when everyone is a rent-seeker, then all receipts are diverted. There is a direct as well as an indirect positive effect of the number of rent-seekers in the economy. Each rent-seeker can transfer individually, hence, bigger the pool of rent-seekers, bigger will be the share of receipts diverted. The indirect effect stems from the effectiveness of the ex-ante constraints. In line with the literature on the theory of deterrence, when the proportion of dishonest individuals is high, it makes the administrative controls less effective.[10]

Second, there is also a negative externality arising from the way the receipts are shared among the rent-seekers, as shown by Tullock (1980), Ehrlich and Lui (1999), and Wadho (2014). The share which an individual gets depends on the size of her unproductive capital relative to that of the aggregate stock of unproductive capital. In other words, an individual's ability to collect rents depends on how large her personal 'power' is relative to that of the others. In this way, the share which she gets not only depends on her own unproductive capital but also on the unproductive capital of others. The share of rents which an individual gets, $\upsilon_{i\tau t}$, is simply equal to her share in the aggregate unproductive capital, that is

$$\upsilon_{i\tau t} = \frac{Q_{i\tau t}}{\sum\limits_{j=1}^{n} Q_{j\tau t}} \tag{7}$$

where $\tau = 1, 2$. Equation (7) implies that the share which an individual gets increases with her investment in the unproductive capital and decreases with the aggregate unproductive capital, as a large aggregate stock of unproductive capital implies a smaller relative capital of an individual.

We further suppose that rent-seeking is illegal and that the rent-seekers run the risk of getting caught and punished. Every rent-seeker faces a probability, z, of being caught. When caught, her entire second period income is confiscated.[11] Parameter z thus reflects the quality of institutions, where better institutions (higher z) implies a higher probability of getting caught and punished. These law enforcement

[8] It seems plausible as, for instance, when individuals are caught, they will not only pay equal to what they were legally supposed to, but they will also pay some penalty on top of that as well.

[9] It may also refer to "the credibility of the government's commitment to policies". θ in our case is the policy variable which entails the government taking away a pre-defined proportion of individuals' income for the transfer purpose. α in this case can be interpreted as a measure of the credibility of government's commitment to its policy since a higher α would imply better administrative controls by the government to prevent leakages in the form of rent-seeking and therefore keeping the government's credibility in tact to redistribute the receipts from market intervention (see Glaeser et al., 2004).

[10] For example, Lui (1986) reports that a fundamental observation on corruption is that it becomes very difficult to audit effectively if too many individuals are corrupt.

[11] The idea is that the government may impose a fine or a penalty as large as an individual's entire second period income to deter rent-seeking. Also, when caught, individuals may be imprisoned for a considerable amount of time, disabling them to work. Furthermore, once an individual is identified as a rent-seeker, then she may lose her job and not to be employed anywhere in the future.

institutions reflect the 'ex-post' institutional constraints which may take the form of policing and legal apparatus of the state which spring into action after rent-seeking is carried out. For simplicity, we assume that the confiscated earnings of rent-seekers are not subject to redistribution (and therefore to appropriation) and are dissipated instead.

3 Individual Decision Problem

Individual decision making involves two dimensions: first, whether to be a rent-seeker or not; and second, the time allocation between schooling, accumulating unproductive capital, and working. They decide between being honest or seeking rents, which depends on the benefits in the form of resisting government intervention, diverting a share from the receipts of this intervention, the behavior of others, and the costs incurred if caught. We proceed to solve the system backwards. In the first step, we obtain the optimal time allocations in both situations, with and without rent-seeking. In the second step, an individual decides to be a rent-seeker or not depending on the utility comparison under two different scenarios; when others do not invest in the unproductive capital, $q_{it} = 0$, and when others invest in the unproductive capital, $q_{it} > 0$.

3.1 High Growth Equilibrium (no rent-seeking)

We start with the decision to invest in the human capital in the high growth equilibrium which is defined as an equilibrium in which $q_{it} = 0$, and the proportion of rent-seeking individuals $d_t = 0$. The maximization problem faced by an individual is

$$\max_{c_{i1t},\, c_{i2t},\, h_{it}} U_{it} = \ln(c_{i1t}) + \beta \ln(c_{i2t})$$

subject to:

$$\begin{aligned}
c_{i1t} &= (1-\theta) y^u_{it} + \frac{\sum_{j=1}^{n} \theta_{j1t} y^u_{jt}}{n} \\
c_{i2t} &= (1-\theta) y^s_{it} + \frac{\sum_{j=1}^{n} \theta_{j2t} y^s_{jt}}{n} \\
y^u_{it} &= (1-h_{it}) \\
y^s_{it} &= \gamma H_{i2t} \\
H_{i2t} &= A H_{i1t} h_{it} \\
0 \leq\ & h_{it} \leq 1
\end{aligned}$$

The first-order condition yields

$$\frac{c_{i2t}}{c_{i1t}} = \gamma \beta A H_{i1t}$$

By substituting in for c_{i1t} and c_{i2t} and by imposing the equilibrium condition, $H_{i2t} = H_{2t}$, the first order conditions yield the time devoted to human capital accumulation in the high-growth equilibrium

$$h^{HG} = \frac{\beta}{1+\beta} \tag{8}$$

From (2), in the high-growth equilibrium, human capital and output grow at

$$1 + g^{HG} = \frac{\beta A}{1+\beta}$$

In the high-growth equilibrium, the growth rate will be higher when the individuals are more patient and when the human capital technology is more productive.[12]

3.2 Low-growth Equilibrium (rent-seeking)

Consider now the case with rent-seeking in which all individuals invest in the unproductive capital, $q_{it} > 0$ $\forall\ i$, and the proportion of rent-seekers $d_t = 1$. The maximization problem faced by an individual is

$$\max_{c_{i1t},\ c_{i2t},\ h_{it},\ q_{it}} \quad U_{it} = \ln(c_{i1t}) + \beta \ln(c_{i2t})$$

subject to:

$$\begin{aligned}
c_{i1t} &= (1-\theta_{i1t})y^u_{it} + \upsilon_{i1t}\sum_{j=1}^{n}\theta_{j1t}y^u_{jt} \\
c_{i2t} &= (1-z)\left[(1-\theta_{i2t})y^s_{it} + \upsilon_{i2t}\sum_{j=1}^{n}\theta_{j2t}y^s_{jt}\right] \\
y^u_{it} &= (1-h_{it}-q_{it}) \\
y^s_{it} &= \gamma H_{i2t} \\
H_{i2t} &= AH_{i1t}h_{it} \\
Q_{it} = Q_{i2t} &= Q_{i1t} = BQ_{i0t}q_{it} \\
\theta_{it} = \theta_{i2t} &= \theta_{i1t} = \theta\left[1-\left(\frac{Q_{i1t}}{\bar{Q}_{1t}}-\alpha\right)\right] \\
\upsilon_{it} = \upsilon_{i2t} &= \upsilon_{i1t} = \frac{Q_{i1t}}{\sum\limits_{i=1}^{n} Q_{i1t}} \\
0 \leq\ &h_{it} \leq 1 \\
0 \leq\ &q_{it} \leq 1
\end{aligned}$$

The first-order condition for h_{it} yields

$$h_{it} = \frac{\beta}{1+\beta}\left[(1-q_{it}) + \frac{\sum_{j=1}^{n}\theta_{jt}\left(1-h_{jt}-q_{jt}\right)}{1-\theta_{it}}\left(\frac{Q_{it}}{\sum\limits_{i=1}^{n}Q_{it}}\right)\right] - \frac{1}{1+\beta}\,\frac{\sum_{j=1}^{n}\theta_{jt}H_{j1t}h_{jt}}{\left(1-\theta_{it}\right)H_{i1t}}\left(\frac{Q_{it}}{\sum\limits_{i=1}^{n}Q_{it}}\right)$$

It is evident that there is a trade-off between the accumulation of human and unproductive capital

[12] The super-script HG represents the high-growth equilibrium.

where more time invested in the accumulation of unproductive capital reduces the investment in human capital.

The first-order condition for q_{it} yields

$$q_{it} = \beta(1-z)\frac{c_{i1t}}{c_{i2t}}\left[\gamma H_{i2t} + \frac{\overline{Q}_t}{\theta\sum\limits_{i=1}^{n} Q_{it}}\sum_{j=1}^{n}\theta_{jt}y^s_{jt}\right] + (1-h_{it}) + \frac{\overline{Q}_t}{\theta\sum\limits_{i=1}^{n} Q_{it}}\sum_{j=1}^{n}\theta_{jt}y^u_{jt} - \frac{\overline{Q}_t}{\theta B Q_{i0t}}(1-\theta_{it})$$

By substituting in for c_{i1t}, c_{i2t}, θ_{it} and by imposing the equilibrium condition, $H_{i2t} = H_{2t}$, $Q_{i1t} = Q_{1t} = Q_{2t}$, the first order conditions yield

$$q^{LG} = \frac{\theta(1+\alpha)}{1+\theta} \tag{9}$$

$$h^{LG} = \frac{\beta}{1+\beta}\left(\frac{1-\alpha\theta}{1+\theta}\right) \tag{10}$$

An increase in the size of government, θ, increases the rewards from rent-seeking thus increasing the investment in unproductive capital.[13] This is primarily because a bigger θ implies: i) a higher return from resisting/evading it, and ii) a larger amount of government receipts that can be appropriated/diverted. Moreover, since there is a trade-off between investment in human and unproductive capital, a bigger size of government reduces the investment in human capital.

Investment in the unproductive capital also increases with the size of ex-anté institutional controls, α. This effect is stemming from individuals' transfer resisting technology whereby an individual ends up paying less than the size of government if her relative unproductive capital is large enough to offset the institutional controls. Thus, bigger the magnitude of α, higher will be the effort required in accumulating the unproductive capital to resist a transfer.[14]

Furthermore, since $\alpha\theta < 1$, equation (9) implies that $q^{LG} < 1$ and hence there is no corner solution where individuals invest their entire time in the accumulation of unproductive capital.

From (2), in the low-growth equilibrium, human capital and output grow at

$$1 + g^{LG} = \frac{\beta A}{1+\beta}\left(\frac{1-\alpha\theta}{1+\theta}\right)$$

There are two important observations to note from the expression for the growth rate given above. The first observation is that since $\frac{1-\alpha\theta}{1+\theta} < 1$, the growth rate with rent-seeking is always lower than the growth rate without rent-seeking. Second, the growth rate decreases with the size of government, θ. However, similar to the case of the absence of rent-seeking, the growth rate with rent-seeking will be higher when individuals are more patient and when the human capital technology is more productive.

3.3 Rent-seeking decision

In the previous section, we presented the individual optimization problem and obtained the optimal time allocation in both human and unproductive capital. In this section, we look at the existence of these

[13] The super-script LG represents the low-growth equilibrium.

[14] This is only true in the case where all individuals are rent-seekers. In any case, when the proportion of rent-seekers is less than one, there will also be a negative effect of α coming through the receipt diverting technology, P_t. In such cases, there can be an increasing or a decreasing effect of α on q_t depending on the size of the two effects.

equilibria where an individual decides to invest in the unproductive capital while comparing her utility in the various scenarios and by taking into account the investment by others. An individual chooses to invest in the unproductive capital and becomes a rent-seeker if it results in a higher level of utility than by not investing. There are two possible scenarios depending on whether others invest in the accumulation of the unproductive capital or not. Since returns to the accumulation of unproductive capital depend on the size of government, θ, therefore it is a crucial element for the existence of equilibrium. Before going into the details of utility comparisons, we suppose that the size of population is sufficiently large, precisely, $\frac{1}{n} \approx 0$. Although the existence of equilibria can be shown for any value of n, making this assumption helps us to explicitly define different thresholds of the size of government which demarcate different equilibria.

3.3.1 When all others do not invest ($q_t = 0$)

Let us first consider the case when all others, $(n-1)$, are not rent-seekers, that is, they do not invest in the unproductive capital. An individual i takes the behavior of all others as given and compares her utility when she also does not invest in the unproductive capital with that when she does. If individual i also opts not to be a rent-seeker, then $q_{it} = 0$ and $h_{it} = h^{HG}$. The utility of being honest is then given by

$$U^h_{iht} = \ln(1 - h^{HG}) + \beta \ln(\gamma A H_{1t} h^{HG}) \tag{11}$$

Throughout the rest of this paper, we use a sub-script to represent the behavior of an individual and a super-script to represent the behavior of all others. We use (r) for rent-seeking and (h) is used to denote honest behavior (no rent-seeking).

Consider now a situation where individual i chooses to be a rent-seeker when everyone else is not. In this case, the proportion of rent-seekers is $d_t = \frac{1}{n}$ and the proportion of government receipts which are diverted is $P_t = \frac{n-(1-\alpha)}{n}$. The first-order conditions yield the following optimal values of time investment in human and unproductive capital

$$h^h_r = \frac{\beta}{1+\beta}\left(\frac{1-\theta\,(1-\alpha)}{1+\theta}\right)$$

$$q^h_r = \frac{\theta(2-\alpha)}{1+\theta}$$

The utility of being a rent-seeker is then given by

$$U^h_{irt} = \ln\left[\left(1 - h^h_r - q^h_r\right) + (2-\alpha)\theta(1-h^{HG})\right] + \beta \ln\left[(1-z)\left(\gamma A H_{1t} h^h_r + (2-\alpha)\theta\gamma A H_{1t} h^{HG}\right)\right] \tag{12}$$

An individual compares her utility in (12) to that in (11), and will be a rent-seeker if the former is greater, otherwise she will not be a rent-seeker.

Proposition 1 $\forall \theta \leq \overline{\theta}$, *there exists an equilibrium (high-growth) where no one is a rent-seeker.*

Proof. See Appendix A. ■

For the size of government lower than $\overline{\theta}$, returns to rent-seeking are so low that no one invests in the unproductive capital. Whereas, for θ greater than $\overline{\theta}$, returns to rent-seeking are high enough that an individual invests in the unproductive capital even when all others do not. This implies that the high-growth equilibrium does not exist for any $\theta > \overline{\theta}$. Where

$$\overline{\theta} = \frac{1 - \mu + \sqrt{(1-\mu)^2 + 4(2-\alpha)\mu(1-\mu)}}{2(2-\alpha)\mu}$$

where $\mu = (1-z)^{\frac{\beta}{1+\beta}}$

The threshold level of government intervention which determines existence of the high-growth equilibrium, $\overline{\theta}$, is endogenous and it varies with individuals' time discounting, β and with the quality of institutions which is represented by α and z. The partial derivatives, $\frac{\partial\overline{\theta}}{\partial\beta}$, $\frac{\partial\overline{\theta}}{\partial z}$, and $\frac{\partial\overline{\theta}}{\partial\alpha}$ are all positive.[15] This implies that when individuals discount their future less and/or there are better quality institutions, the high-growth equilibrium can be compatible with a bigger size of the government. Interestingly, in the absence of ex-post constraints, that is when $z = 0$, then $\overline{\theta} = 0$ and the high-growth equilibrium does not exist. This is very intuitive as with weaker institutions, where there is no chance of being detected and penalized for rent-seeking, the returns to being a rent-seeker will always exceed returns to being honest. In the other extreme when an agent is certain of getting caught, that is when $z = 1$, then $\overline{\theta} = \infty$ and for any size of the government, it will always pay an individual to remain honest.

3.3.2 When all others invest ($q_t = q^{LG} > 0$)

Consider now the case when all other $(n-1)$ individuals are rent-seekers, that is, they invest in the unproductive capital. An individual i takes the behavior of all others as given and compares her utility for the case when she also invests in the unproductive capital with that when she does not. If individual i also opts to be a rent-seeker, then $q_{it} = q^{LG}$ and $h_{it} = h^{LG}$. The utility of being a rent-seeker is given by

$$U^r_{irt} = \ln(1 - h^{LG} - q^{LG}) + \beta \ln((1-z)\gamma A H_{1t} h^{LG}) \tag{13}$$

When all others are rent-seekers and individual i deviates and remains honest, then in this case the proportion of rent-seekers is $d_t = \frac{n-1}{n}$. Individual i invests $h_{it} = h^{HG} = \frac{\beta}{1+\beta}$ and her utility is then given by

$$U^r_{iht} = \ln((1-\theta)(1-h^{HG})) + \beta \ln((1-\theta)\gamma A H_{1t} h^{HG}) \tag{14}$$

An individual compares her utility in (14) to that in (13), and will be rent-seeker if the latter is greater, otherwise she will be honest.

Proposition 2 $\forall\theta \geq \underline{\theta}$, *there exists an equilibrium (low-growth) where everyone is a rent-seeker.*

Proof. See Appendix B. ■

When the size of government is greater than the threshold $\underline{\theta}$, returns to rent-seeking are so high that it will always pay to invest in the unproductive capital and be a rent-seeker. Whereas, for $\theta < \underline{\theta}$, an individual opts not to be rent-seeker even when all others are rent-seekers. This implies that the low-growth equilibrium does not exist for $\theta < \underline{\theta}$. Where

[15] For derivatives, see Appendix A.

$$\underline{\theta} = \frac{\alpha\mu + \sqrt{(\alpha\mu)^2 + 4(1-\mu)}}{2}$$

recall that $\mu = (1-z)^{\frac{\beta}{1+\beta}}$

The threshold $\underline{\theta}$ is endogenous and it varies with individuals' time discounting, β and the quality of institutions represented by α and z. The partial derivatives, $\frac{\partial\underline{\theta}}{\partial\beta}$, $\frac{\partial\underline{\theta}}{\partial z}$, and $\frac{\partial\underline{\theta}}{\partial\alpha}$ are all positive.[16] This implies that when individuals discount their future less and/or there are better quality institutions, then the range of the size of government for which the low-growth equilibrium exists decreases. In the absence of ex-post institutional controls, that is when $z = 0$, then $\underline{\theta} = \alpha$ and the low-growth equilibrium exists for any size of government greater than α. Intuitively, in the absence of any ex-post cost, rent-seeking is profitable only if the size of government is large enough to cover the ex-anté cost of rent-seeking. In another extreme when $z = 1$, there is no low-growth equilibrium even when the entire economy is run by the government, i.e. $\theta = 1$.

Proposition 1 and 2 both suggest that the growth regime (high or low) depends on the size of government, and the thresholds of the size of government which demarcate these equilibria are endogenous and crucially, depend on individuals' time discounting. Next, we compare the two thresholds and the role played by individuals' time discounting. To proceed further, we abstain from boundary situations and suppose that $\varepsilon \leq \theta \leq \omega$ where $\varepsilon \to 0$ and $\omega \to 1$.

Proposition 3 $\forall\overline{\overline{\beta}} \leq \beta \leq 1$, *there exists only a unique high-growth equilibrium* $\forall\theta \leq \omega \to 1$. $\forall\widetilde{\beta} \leq \beta \leq \underline{\underline{\beta}}$, *there exists only a unique low-growth equilibrium* $\forall\theta \geq \varepsilon \to 0$. *And* $\forall\underline{\underline{\beta}} < \beta < \overline{\overline{\beta}}$ *there are multiple equilibria.*

Proof. See Appendix C. ■

$$\overline{\overline{\beta}} = \frac{\ln\left(\frac{1-\alpha\omega}{1-\omega^2}\right)}{\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]}, \text{ and } \underline{\underline{\beta}} = \frac{\ln\left(\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right)}{\ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]}$$

Our results in Proposition 3 show that there exists a non-monotonic relationship between the size of government and growth outcomes. When the level of individuals' patience is either very high or very low, there is always a unique equilibrium with high growth in case of the former and low growth in the case of the latter. However, the uniqueness of the high-growth equilibrium requires that the quality of institutions is substantially high when β is bounded from the above at 1. Specifically, it requires that the ex-anté institutional controls $\alpha > \overline{\alpha} = \frac{\omega^2 - z}{(1-z)\omega}$. Both equilibrium regimes are stable and in the case of the low-growth equilibrium, the growth rate decreases with the size of government. When individuals' level of patience is in the intermediate range, there are multiple equilibria and growth outcomes depend on the size of government. When the level of patience is less than $\overline{\overline{\beta}}$ but greater than $\overline{\beta} = \frac{\ln\left(\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right)}{\ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]}$, there is a unique high-growth equilibrium for a smaller size of government ($\theta < \underline{\theta}$), and both the high-growth and the low-growth equilibrium co-exist for a larger size of government ($\underline{\theta} \leq \theta \leq \omega$).

FIGURE 1 HERE

[16] For derivatives, see Appendix B.

On the other hand, when individuals' level of patience is greater than $\underline{\underline{\beta}}$ but less than $\underline{\beta} = \frac{\ln\left(\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right)}{\ln\left[\frac{1-\varepsilon^2}{(1-z)[1-\alpha\varepsilon]}\right]}$, there are two growth regimes depending on the size of government. For a smaller size of government ($\theta < \overline{\theta}$), both the high-growth and the low-growth equilibrium co-exist, and for a larger size of government ($\overline{\theta} \leq \theta \leq \omega$), there is a unique low-growth equilibrium.

FIGURE 2 HERE

Finally, when the level of individuals' patience is between $\underline{\beta}$ and $\overline{\beta}$, there is a unique high-growth equilibrium for a smaller size of government ($\theta < \underline{\theta}$), a unique low-growth equilibrium for a larger size of government ($\theta > \overline{\theta}$), and for the intermediate range ($\underline{\theta} < \theta < \overline{\theta}$), both the high-growth and the low-growth equilibrium co-exist.

FIGURE 3 HERE

In all of the specifications, better quality of institutions (represented by z) implies an increase in the range of the size of government for which there is the high-growth equilibrium and a decrease in the range of the size of government for which there is the low-growth equilibrium. Furthermore, in the case of the low-growth equilibrium, the growth rate will always decrease with the size of government.

4 Patience thresholds and equilibria

In the previous section, we demonstrated that the steady-state levels of equilibrium are influenced by the size of government. More importantly, the thresholds of size of government which demarcate the high and the low growth steady-states are endogenous and crucially, depend on individuals' discounting of the future. In this section, we go one step ahead and model the discount factor, β, as endogenous. We endogenize discounting based on three premises highlighted in the recent research on time discounting or patience. Firstly, a positive correlation is reported between education and low discounting of future (high patience). Harrison et al. (2002) find that skilled individuals, where the skill-level is proxied by the level of education, have a significantly low discount rate when compared to that of the unskilled individuals. Similar results are obtained by Bauer and Chytilova (2010) who use the data from Ugandan villages, and by Kirby et al. (2002) using the data on Tsimane Amerindians of the Bolivian rainforest. Secondly, there is a positive correlation between parents' rate of time preference and that of their children. Brown and van der Pol (2015) find a positive correlation between the intergenerational rate of time preference using the data from an Australian household survey. The same is observed by Webley and Nyhus (2006) for a Dutch panel data. Thirdly, there is a positive correlation between parental education and a lower discounting of future by their children. Kirby et al. (2002) find that individuals' discount rate decreases with parental (father's) education. Bjorklund and Salvanes (2011, ch:3) indicate that individuals' incentive to accumulate human capital is influenced by the stock of their parents' human capital.

Based on the above findings, we assume that more educated parents discount future less and they also transfer the same trait to their next generation. Specifically, an individual's level of patience, β_t is a positive function of her parental education. Since, all individuals within a generation are identical, therefore, the level of patience for any generation is a function of their average initial stock of human capital, H_{1t}. Additionally, we follow the convention used in the economic models of inter-temporal decision-making whereby individuals' level of patience is modeled as strictly positive and bounded from above at one (see Lucas, 1988; Galor and Zeira, 1993; Redding, 1996; Eicher et al., 2009 for examples).

$$\beta_t = \beta\left(\frac{\sum_{i=1}^{n} H_{i1t}}{n}\right) = \beta\left(H_{1t}\right) = \beta_0 H_{1t}^{\lambda} \tag{16}$$

where $\widetilde{\beta} \leq \beta_0 H_{1t}^{\lambda} \leq 1$ and $0 \leq \lambda \leq 1$.

Proposition 4 *$\forall H_{1t} \geq \overline{\overline{H}} \Longrightarrow \overline{\overline{\beta}} \leq \beta_t \leq 1$, there exists only a unique high-growth equilibrium $\forall \theta \leq \omega \to 1$. $\forall H_{1t} \leq \underline{\underline{H}} \Longrightarrow \widetilde{\beta} \leq \beta_t \leq \underline{\underline{\beta}}$, there exists only a unique low-growth equilibrium $\forall \theta \geq \varepsilon \to 0$. And $\forall \underline{\underline{H}} < H_{1t} < \overline{\overline{H}}$ there are multiple equilibria.*

Proof. See Appendix D. ■

$$\overline{\overline{H}} = \left(\frac{\ln\left(\frac{1-\alpha\omega}{1-\omega^2}\right)}{\beta_0 \ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]}\right)^{\frac{1}{\lambda}}, \text{ and } \underline{\underline{H}} = \left(\frac{\ln\left(\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right)}{\beta_0 \ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]}\right)^{\frac{1}{\lambda}}$$

The results in Proposition 4 follow from our results in Proposition 3. There is a non-monotonic relationship between the size of government and growth outcomes. There is path-dependence in growth outcomes as the equilibrium regime depends on the level of inherited human capital. When the level of inherited human capital is very high, there is a unique high-growth equilibrium which does not depend on the size of government. Conversely, when the level of inherited human capital is very low, there is a unique low-growth equilibrium which does not depend on the size of government. However, in this case, the growth rate decreases with the size of government. Whereas for intermediate levels of the inherited human capital, there are multiple equilibria and the equilibrium (high or low) depends on the size of government. In all of the cases, in the low-growth regime, the growth rate decreases with the size of government.

Path-dependent growth outcomes with threshold effects is qualitatively similar to Azariadis and Drazen (1990). Our results, though, differ from theirs in a number of ways. First, in the settings of our model, rather than affecting the productivity of the human capital accumulation technology, the level of inherited human capital affects individuals' patience, which makes investment in the productive human capital with delayed realization more valuable. This is in line with the threshold effects highlighted in the recent empirical literature. For example, Haveman et al. (1991) and Manski et al. (1992) find that parental completion of high school and one or two years of post-secondary schooling typically has a larger effect on children's schooling when compared to other levels of parental education. Similarly, Hryshko et al. (2011) report that individuals' risk aversion exhibits threshold effects in the level of parental education. They find that parental education beyond grade 11 has a significant impact on an individual's risk aversion, whereas, Dohmen et al. (2010) find that there is a significant and a robust relationship between individuals' risk aversion and their impatience. Second, as opposed to being exogenous in Azariadis and Drazen (1990), the thresholds of inherited human capital which demarcate the equilibria are endogenous and depend on quality of institutions represented by α and z. Both $\overline{\overline{H}}$ and $\underline{\underline{H}}$ decrease with an improvement in the ex-anté institutional quality, α, as well as with the ex-post institutional quality, z.[17] This implies that in comparison to countries with poor institutional quality (low α and/or z), the ones with better quality institutions (high α and/or z) will be in the high-growth steady-state equilibrium with relatively smaller levels of the inherited human capital.

The main qualitative difference between the results in Proposition 4 (with endogenous time discounting) with the results in Proposition 3 (with exogenous time discounting) and Azariadis and Drazen (1990) is that in the case of endogenous discounting, the economy will endogenously switch growth regimes once the level of human capital reaches the critical level required for that particular regime to exist. From equation (10), the time invested in the accumulation of human capital increases with individuals' level of patience which in turn increases with the level of their inherited human capital. However, the time it takes for an economy to endogenously switch growth regimes crucially depends on the size of government as well as on the quality of institutions. Economies with bigger size of government and/or with poor quality institutions will take a relatively longer time to switch to the high-growth equilibrium. The size of government, in fact, affects both the growth rate which is given by equation (10) and the existence of equilibria given in Proposition 4.

When the economy endogenously switches regimes in Proposition 4, there will only be one stable growth regime. The quality of institutions represented by the ex-anté institutional constraints, α, then determine if the stable equilibrium is the unique high-growth equilibrium or both the high and the low growth equilibrium are stable. Consider first the case when ex-anté institutional controls are not extremely high, specifically if $\alpha < \overline{\alpha} = \frac{\omega^2 - z}{(1-z)\omega}$. In this case $\overline{\overline{\beta}} > 1$, and since β is bounded from above at 1, the growth regime in Proposition 4 in which there is only a unique high-growth equilibrium does not exist. The economy will eventually switch to an equilibrium regime with $\beta = 1$ in which both the high and the low growth equilibrium are stable. And since $\beta = 1 < \overline{\overline{\beta}}$, from Proposition 4 we have

FIGURE 1 HERE

In the case of high-growth equilibrium, the economy grows at $g^{HG} = \frac{A}{2} - 1$, and in the case of the low growth-equilibrium, the economy grows at $g^{LG} = \frac{A}{2}\left(\frac{1-\alpha\theta}{1+\theta}\right) - 1$, and $g^{LG} < g^{HG}$.

In the case when ex-anté institutional constraints are extremely high, specifically when $\alpha \geq \overline{\alpha} = \frac{\omega^2 - z}{(1-z)\omega}$. In this case, $\overline{\overline{\beta}} \leq 1$. From Proposition 4, the only stable growth regime is the one in which there exists only a unique high-growth equilibrium. The economy will eventually converge to this equilibrium and there is no stable low-growth equilibrium. Note that for $\omega \to 1$, the threshold $\overline{\alpha}$ is close to 1, thus the case in which the unique high-growth equilibrium is the only stable equilibrium is rather an extreme case. A more normal case is the one in which $\alpha < \overline{\alpha}$ and both the high-growth, and the low-growth equilibrium are stable. The equilibrium then is determined by the size of government.

[17] For derivatives, see Appendix D.

5 Conclusion

This research explores the relationship between the size of government and economic growth in the presence of both a productive and an unproductive capital. Consistent with the recent empirical findings, we model individuals' time discounting as an endogenous function of their inherited (or parental) human capital. High level of inherited human capital implies lower discounting of future, which promotes investment in human capital. We show that with endogenous discounting, growth outcomes are path-dependent and depend on the level of inherited human capital. For very high and very low level of the inherited human capital, there is a unique high-growth and a unique low-growth equilibrium, respectively. In both cases, the equilibrium does not depend on the size of government. For intermediate levels of the inherited human capital, there are three different equilibria, each of which depends on the size of government. The thresholds of inherited human capital which demarcate different equilibria are endogenous and positively depend on the quality of institutions. However, there will only be one stable growth regime and the economy will endogenously converge to this regime. Which of the regimes is stable is determined by the quality of institutions. When ex-anté institutional constraints are not extremely high, the stable growth regime is the one in which both the high and the low growth equilibrium exist. The high-growth equilibrium exists for a smaller size of the government and beyond this threshold both the high-growth and the low-growth equilibrium coexist. When ex-anté institutional constraints are extremely high, then the growth regime with a unique high-growth equilibrium is the only stable regime.

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Appendix A

An individual compares her utility of being honest given in (11) with that of being a rent-seeker given in (12). We show that $\forall\theta \leq \overline{\theta}$, the utility of being honest is greater and the individual does not become a rent-seeker. Since all individuals are identical, in the equilibrium no one invests in rent-seeking and the economy is in the high-growth equilibrium.

$$U_{iht}^{h}(\gamma, A, H_{i1t}, \beta) \geq U_{irt}^{h}(\gamma, A, H_{i1t}, \beta, \theta, \alpha, z)$$

$$\begin{aligned}&\ln(1-h^{HG}) + \beta\ln(\gamma A H_{1t} h^{HG}) \geq \\ &\qquad \ln\left[\left(1-h_r^h-q_r^h\right)+(2-\alpha)\theta(1-h^{HG})\right] + \beta\ln\left[(1-z)\left(\gamma A H_{1t} h_r^h + (2-\alpha)\theta\gamma A H_{1t} h^{HG}\right)\right]\end{aligned}$$

By rearranging,

$$\begin{aligned}&\ln(1-h^{HG}) + \beta\ln(\gamma A H_{1t} h^{HG}) \geq \\ &\qquad \ln\left[\left(1-h_r^h-q_r^h\right)+(2-\alpha)\theta(1-h^{HG})\right] + \beta\ln\left[(1-z)\left(\gamma A H_{1t} h_r^h + (2-\alpha)\theta\gamma A H_{1t} h^{HG}\right)\right]\end{aligned}$$

By substituting in for h^{HG}, h_r^h, and q_r^h

$$\begin{aligned}&\ln\left(\frac{1}{1+\beta}\right) + \beta\ln\left(\frac{\beta\gamma A H_{1t}}{1+\beta}\right) \geq \\ &\ln\left[\frac{1}{1+\beta}\left(\frac{1-\theta\,(1-\alpha)}{1+\theta}\right)+\frac{(2-\alpha)\theta}{1+\beta}\right] + \beta\ln\left[(1-z)\left(\frac{\beta\gamma A H_{1t}}{1+\beta}\right)\left(\left(\frac{1-\theta\,(1-\alpha)}{1+\theta}\right)+(2-\alpha)\theta\right)\right]\end{aligned}$$

$$\begin{aligned}&\ln\left(\frac{1}{1+\beta}\right) + \beta\ln\left(\frac{\beta\gamma A H_{1t}}{1+\beta}\right) \geq \\ &\qquad\qquad \ln\left(\frac{1+\theta+(2-\alpha)\theta^2}{(1+\beta)(1+\theta)}\right) + \beta\ln\left[\frac{\beta\gamma A H_{1t}(1-z)}{1+\beta}\left(\frac{1+\theta+(2-\alpha)\theta^2}{1+\theta}\right)\right]\end{aligned}$$

$$\ln\left(\frac{1+\theta}{1+\theta+(2-\alpha)\theta^2}\right) - \frac{\beta}{1+\beta}\ln(1-z) \geq 0$$

By taking anti-log and solving for θ, we get

$$\theta \leq \frac{1-\mu+\sqrt{(1-\mu)^2+4(2-\alpha)\mu(1-\mu)}}{2(2-\alpha)\mu} \equiv \overline{\theta}$$

where $\mu = (1-z)^{\frac{\beta}{1+\beta}}$. Since $0 \leq \theta \leq 1$, we ignore the negative root.

Now, taking the partial derivatives of $\overline{\theta}$ with respect to β, z, and α.

$$\frac{\partial\overline{\theta}}{\partial\beta} = \frac{\partial\overline{\theta}}{\partial\mu}\frac{\partial\mu}{\partial\beta}$$

$$\frac{\partial\overline{\theta}}{\partial\mu} = \frac{1+3\mu-2\alpha\mu+\sqrt{(1-\mu)^2+4(2-\alpha)\mu(1-\mu)}}{-2(2-\alpha)\mu^2\sqrt{(1-\mu)^2+4(2-\alpha)\mu(1-\mu)}} < 0$$

$$\frac{\partial\mu}{\partial\beta} = \frac{\partial(1-z)^{\frac{\beta}{1+\beta}}}{\partial\beta} = \frac{(1-z)^{\frac{\beta}{1+\beta}}\ln(1-z)}{(1+\beta)^2} < 0 \text{ since } \ln(1-z) < 0$$

Hence

$$\frac{\partial\overline{\theta}}{\partial\beta} > 0$$

Whereas

$$\frac{\partial\overline{\theta}}{\partial z} = \frac{\partial\overline{\theta}}{\partial\mu}\frac{\partial\mu}{\partial z}$$

$$\frac{\partial(1-z)^{\frac{\beta}{1+\beta}}}{\partial z} = -\beta\frac{(1-z)^{\frac{\beta}{1+\beta}}}{(1+\beta)(1-z)} < 0$$

Hence

$$\frac{\partial\overline{\theta}}{\partial z} > 0$$

And

$$\frac{\partial\overline{\theta}}{\partial\alpha} = \frac{(1-\mu)\left[1+3\mu-2\alpha\mu+\sqrt{(1-\mu)^2+4(2-\alpha)\mu(1-\mu)}\right]}{2(2-\alpha)^2\mu\sqrt{(1-\mu)^2+4(2-\alpha)\mu(1-\mu)}} > 0$$

Appendix B

An individual compares her utility of being a rent-seeker given in (13) with that of being honest given in (14). We show that $\forall\theta \geq \underline{\theta}$, the utility of being a rent-seeker is greater and the individual becomes a rent-seeker. Since all individuals are identical, in the equilibrium everyone invests in rent-seeking and the economy is in the low-growth equilibrium.

$$U^r_{irt}(\gamma, A, H_{i1t}, \beta, \theta, \alpha, z) \geq U^r_{iht}(\gamma, A, H_{i1t}, \beta)$$

$$\ln(1-h^{LG}-q^{LG}) + \beta\ln((1-z)\gamma AH_{1t}h^{LG}) \geq \ln((1-\theta)(1-h^{HG})) + \beta\ln((1-\theta)\gamma AH_{1t}h^{HG})$$

By substituting in for h^{LG}, q^{LG}, and h^{HG}

$$\ln\left(\frac{1}{1+\beta}\left(\frac{1-\alpha\theta}{1+\theta}\right)\right) + \beta\ln\left(\frac{\beta\gamma AH_{1t}(1-z)}{1+\beta}\left(\frac{1-\alpha\theta}{1+\theta}\right)\right) \geq \ln\left(\frac{1-\theta}{1+\beta}\right) + \beta\ln\left(\frac{\beta\gamma AH_{1t}(1-\theta)}{1+\beta}\right)$$

$$\ln\left(\frac{1-\theta^2}{1-\alpha\theta}\right) - \frac{\beta}{1+\beta}\ln(1-z) \leq 0$$

By taking anti-log and solving for θ, we get

$$\theta \geq \frac{\alpha\mu + \sqrt{(\alpha\mu)^2 + 4(1-\mu)}}{2} \equiv \underline{\theta}$$

where $\mu = (1-z)^{\frac{\beta}{1+\beta}}$. Since $0 \leq \theta \leq 1$, we ignore the negative root.

Now, taking the partial derivatives of $\underline{\theta}$ with respect to β, z, and α.

$$\frac{\partial\underline{\theta}}{\partial\beta} = \frac{\partial\underline{\theta}}{\partial\mu}\frac{\partial\mu}{\partial\beta}$$

We know from Appendix A that $\frac{\partial\mu}{\partial\beta} < 0$, and $\frac{\partial\underline{\theta}}{\partial\mu} = \frac{-2+\alpha^2\mu+\alpha\sqrt{(\alpha\mu)^2+4(1-\mu)}}{2\sqrt{(\alpha\mu)^2+4(1-\mu)}} < 0$, we have $\frac{\partial\underline{\theta}}{\partial\beta} > 0$. Similarly, $\frac{\partial\underline{\theta}}{\partial z} = \frac{\partial\underline{\theta}}{\partial\mu}\frac{\partial\mu}{\partial z} > 0$. Whereas, $\frac{\partial\underline{\theta}}{\partial\alpha} = \frac{\mu\left[\alpha\mu+\sqrt{(\alpha\mu)^2+4(1-\mu)}\right]}{2\sqrt{(\alpha\mu)^2+4(1-\mu)}} > 0$

Appendix C

In this appendix, we prove Proposition.3. The conditions for the existence of the high-growth and the low-growth equilibrium are given in Proposition.1 and Proposition.2, respectively. An economy will be in the high growth equilibrium $\forall\theta \leq \overline{\theta} = \frac{1-\mu+\sqrt{(1-\mu)^2+4(2-\alpha)\mu(1-\mu)}}{2(2-\alpha)\mu}$, and it will be in the low-growth equilibrium $\forall\theta \geq \underline{\theta} = \frac{\alpha\mu+\sqrt{(\alpha\mu)^2+4(1-\mu)}}{2}$ and $\varepsilon \leq \theta \leq \omega$ where $\varepsilon \to 0$ and $\omega \to 1$.

(i) First we show that when the level of patience $\overline{\overline{\beta}} \leq \beta \leq 1$, then there is a unique high-growth equilibrium. The proof is based on two pieces; a) the high-growth equilibrium exists for all $\varepsilon \leq \theta \leq \omega$, and b) the low-growth equilibrium does not exist for all $\varepsilon \leq \theta \leq \omega$.

a) the high-growth equilibrium exists for all $\varepsilon \leq \theta \leq \omega$ when $\overline{\theta} \geq \omega$

$$\overline{\theta} \geq \omega$$

$$\frac{1-\mu+\sqrt{(1-\mu)^2+4(2-\alpha)\mu(1-\mu)}}{2(2-\alpha)\mu} \geq \omega$$

$$\mu \leq \frac{1+\omega}{1+\omega+(2-\alpha)\,\omega^2}$$

By substituting in for $\mu = (1-z)^{\frac{\beta}{1+\beta}}$

$$(1-z)^{\frac{\beta}{1+\beta}} \leq \frac{1+\omega}{1+\omega+(2-\alpha)\,\omega^2}$$

By solving for β

$$\beta \geq \frac{\ln\left(\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right)}{\ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} \equiv \overline{\beta} \tag{C-1}$$

b) the low-growth equilibrium does not exist for all $\varepsilon \leq \theta \leq \omega$ when $\underline{\theta} \geq \omega$

$$\underline{\theta} \geq \omega$$

$$\frac{\alpha\mu+\sqrt{(\alpha\mu)^2+4(1-\mu)}}{2} \geq \omega$$

$$\mu \leq \frac{1-\omega^2}{1-\alpha\omega}$$

By substituting in for $\mu = (1-z)^{\frac{\beta}{1+\beta}}$

$$(1-z)^{\frac{\beta}{1+\beta}} \leq \frac{1-\omega^2}{1-\alpha\omega}$$

By solving for β

$$\beta \geq \frac{\ln\left(\frac{1-\alpha\omega}{1-\omega^2}\right)}{\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]} \equiv \overline{\overline{\beta}} \tag{C-2}$$

where for $\overline{\overline{\beta}} \leq 1$ the following condition must hold: $\frac{\omega^2-z}{(1-z)\omega} \leq \alpha \leq \omega$.

By comparing the two thresholds of patience above in (a) and (b), we show that $\overline{\overline{\beta}} \geq \overline{\beta}$. This implies that if (b) is true, then (a) will always be true. Hence $\forall\overline{\overline{\beta}} \leq \beta \leq 1$, there is a unique high-growth equilibrium given that $\alpha \geq \overline{\alpha} = \frac{\omega^2 - z}{(1-z)\omega}$

$$\overline{\overline{\beta}} \geq \overline{\beta}$$

$$\frac{\ln\left(\frac{1-\alpha\omega}{1-\omega^2}\right)}{\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]} \geq \frac{\ln\left(\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right)}{\ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]}$$

$$\alpha \leq \frac{\omega(2\omega-1)}{1-\omega(1-\omega)}$$

Given that $0 \leq \alpha \leq 1$ and $\omega \to 1$, the condition above will always hold.

(ii) Second, we show that when the level of patience $\widetilde{\beta} \leq \beta \leq \underline{\underline{\beta}}$, then there is a unique low-growth equilibrium. The proof is based on two pieces; a) the low-growth equilibrium exists for all $\varepsilon \leq \theta \leq \omega$, and b) the high-growth equilibrium does not exist for all $\varepsilon \leq \theta \leq \omega$.

a) the low-growth equilibrium exists for all $\varepsilon \leq \theta \leq \omega$ when $\underline{\theta} \leq \varepsilon$

$$\underline{\theta} \leq \varepsilon$$

$$\frac{\alpha\mu + \sqrt{(\alpha\mu)^2 + 4(1-\mu)}}{2} \leq \varepsilon$$

$$\mu \geq \frac{1-\varepsilon^2}{1-\alpha\varepsilon}$$

By substituting in for $\mu = (1-z)^{\frac{\beta}{1+\beta}}$

$$(1-z)^{\frac{\beta}{1+\beta}} \geq \frac{1-\varepsilon^2}{1-\alpha\varepsilon}$$

By solving for β

$$\beta \leq \frac{\ln\left(\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right)}{\ln\left[\frac{1-\varepsilon^2}{(1-z)[1-\alpha\varepsilon]}\right]} \equiv \underline{\beta} \tag{C-3}$$

b) the high-growth equilibrium does not exist for all $\varepsilon \leq \theta \leq \omega$ when $\overline{\theta} \leq \varepsilon$

$$\overline{\theta} \leq \varepsilon$$

$$\frac{1-\mu+\sqrt{(1-\mu)^2 + 4(2-\alpha)\mu(1-\mu)}}{2(2-\alpha)\mu} \leq \varepsilon$$

$$\mu \geq \frac{1+\varepsilon}{1+\varepsilon+(2-\alpha)\varepsilon^2}$$

By substituting in for $\mu = (1-z)^{\frac{\beta}{1+\beta}}$

$$(1-z)^{\frac{\beta_t}{1+\beta_t}} \geq \frac{1+\varepsilon}{1+\varepsilon+(2-\alpha)\varepsilon^2}$$

By solving for β

$$\beta \leq \frac{\ln\left(\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right)}{\ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} \equiv \underline{\underline{\beta}} \tag{C-4}$$

By comparing the two thresholds of patience in (a) and (b), we show that $\underline{\underline{\beta}} \leq \underline{\beta}$. This implies that if

(b) is true, then (a) will always be true. Hence $\forall\widetilde{\beta} \leq \beta \leq \underline{\underline{\beta}} \implies \overline{\theta}, \underline{\theta} \leq \varepsilon$, there is a unique low-growth equilibrium.

$$\underline{\underline{\beta}} \leq \underline{\beta}$$
$$\frac{\ln\left(\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right)}{\ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} \leq \frac{\ln\left(\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right)}{\ln\left[\frac{1-\varepsilon^2}{(1-z)[1-\alpha\varepsilon]}\right]}$$
$$\alpha \geq \frac{-\varepsilon(1-2\varepsilon)}{1-\varepsilon(1-\varepsilon)}$$

Given that $0 \leq \alpha \leq 1$ and $\varepsilon \to 0$, the condition above will always hold.

(iii) Third, we show that when the level of patience $\overline{\overline{\beta}} > \beta > \underline{\underline{\beta}}$, then there are multiple equilibria. In this case, there are three sub-cases, a) $\overline{\overline{\beta}} > \beta > \overline{\beta}$, b) $\overline{\beta} > \beta > \underline{\beta}$, and c) $\underline{\beta} > \beta > \underline{\underline{\beta}}$.

a) $\forall\overline{\overline{\beta}} > \beta > \overline{\beta} \implies \overline{\theta} > \omega$, and $\varepsilon \leq \underline{\theta} < \omega$ (this also requires $\overline{\beta} \geq \underline{\beta}$). There will be a unique high-growth equilibrium $\forall\theta < \underline{\theta}$, and for $\underline{\theta} \leq \theta \leq \omega$ both the high-growth and the low-growth equilibrium coexist.

[Figure 1 here]

We also show that $\overline{\beta} \geq \underline{\beta}$

$$\overline{\beta} \geq \underline{\beta}$$
$$\frac{\ln\left(\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right)}{\ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} \geq \frac{\ln\left(\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right)}{\ln\left[\frac{1-\varepsilon^2}{(1-z)[1-\alpha\varepsilon]}\right]}$$
$$\alpha \leq \frac{2\omega^2(1-\varepsilon^2) - \varepsilon^2(1+\omega)}{\omega^2(1-\varepsilon^2) - \varepsilon(1+\omega)}$$

Since $\varepsilon < 1$ and $0 \leq \alpha \leq 1$, this always holds.

b) $\forall\overline{\beta} > \beta > \underline{\beta} \implies \varepsilon \leq \overline{\theta} < \omega$, and $\varepsilon \leq \underline{\theta} < \omega$. There will be a unique high-growth equilibrium $\forall\theta < \underline{\theta}$, a unique low-growth equilibrium $\forall\theta > \overline{\theta}$ and for $\underline{\theta} \leq \theta \leq \overline{\theta}$ both the high-growth and the low-growth equilibrium coexist.

[Figure 2 here]

c) $\forall\underline{\beta} > \beta > \underline{\underline{\beta}} \implies \underline{\theta} \leq \varepsilon$, and $\varepsilon \leq \overline{\theta} < \omega$. There will be a unique low-growth equilibrium $\forall\theta > \overline{\theta}$, and for $\varepsilon \leq \theta \leq \overline{\theta}$ both the high-growth and the low-growth equilibrium coexist.

[Figure 3 here]

(iv) Partial derivatives of the patience thresholds with respect to z and α

Partial derivatives of $\underline{\underline{\beta}}$ with respect to z and α.

$$\frac{\partial\underline{\underline{\beta}}}{\partial z} = \frac{-1}{(1-z)\ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} < 0$$
$$\frac{\partial\underline{\underline{\beta}}}{\partial \alpha} = \frac{-\varepsilon^2}{[1+\varepsilon+(2-\alpha)\varepsilon^2]}\left[\frac{1}{\ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} + \frac{1}{\ln\left[\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right]}\right] < 0$$

Taking partial derivatives of $\overline{\overline{\beta}}$ with respect to z and α.

$$\frac{\partial\overline{\overline{\beta}}}{\partial z} = \frac{-1}{(1-z)\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]} < 0$$

$$\frac{\partial\overline{\overline{\beta}}}{\partial \alpha} = \frac{-\omega}{(1-\alpha\omega)}\left[\frac{1}{\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]} + \frac{1}{\ln\left[\frac{1-\alpha\omega}{1-\omega^2}\right]}\right] < 0$$

Taking partial derivatives of $\underline{\beta}$ with respect to z and α.

$$\frac{\partial\underline{\beta}}{\partial z} = \frac{-1}{(1-z)\ln\left[\frac{1-\varepsilon^2}{(1-z)(1-\alpha\varepsilon)}\right]} < 0$$

$$\frac{\partial\underline{\beta}}{\partial \alpha} = \frac{-\varepsilon}{(1-\alpha\varepsilon)}\left[\frac{1}{\ln\left[\frac{1-\varepsilon^2}{(1-z)(1-\alpha\varepsilon)}\right]} + \frac{1}{\ln\left[\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right]}\right] < 0$$

Taking partial derivatives of $\overline{\beta}$ with respect to z and α.

$$\frac{\partial\overline{\beta}}{\partial z} = \frac{-1}{(1-z)\ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} < 0$$

$$\frac{\partial\overline{\beta}}{\partial \alpha} = \frac{-\omega^2}{[1+\omega+(2-\alpha)\,\omega^2]}\left[\frac{1}{\ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} + \frac{1}{\ln\left[\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right]}\right] < 0$$

Appendix D

In this appendix, we prove Proposition 4. The thresholds of patience demarcating different equilibrium regimes are given in Proposition 3. By treating individuals' level of patience as endogenous as expressed in equation (15), here we find the corresponding thresholds of the stock of inherited human capital and the equilibrium regimes which they demarcate. We also perform the comparative static analysis here.

(i) First we show that when the level of inheried human capital $H_{1t} \geq \overline{\overline{H}} \implies \overline{\overline{\beta}} \leq \beta_t \leq 1$, then there is a unique high-growth equilibrium.

a) Using (C-1), by substituting in for β_t from (15), and by solving for H_{1t} we get

$$H_{1t} \geq \left(\frac{\ln\left(\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right)}{\beta_0\ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]}\right)^{\frac{1}{\lambda}} \equiv \overline{H}$$

where the threshold $\overline{H}$ of inherited human capital corresponds to the threshold $\overline{\beta}$ of patience in Appendix C (i) a). Based on our results in Appendix C, this implies that $\forall H_{1t} \geq \overline{H}$ the high-growth equilibrium exists for all $\varepsilon \leq \theta \leq \omega$.

b) Using (C-2), by substituting in for β_t from (15), and by solving for H_{1t} we get

$$H_{1t} \geq \left(\frac{\ln\left(\frac{1-\alpha\omega}{1-\omega^2}\right)}{\beta_0\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]}\right)^{\frac{1}{\lambda}} \equiv \overline{\overline{H}}$$

The threshold $\overline{\overline{H}}$ of inherited human capital corresponds to the threshold $\overline{\overline{\beta}}$ of patience in (C-2). Where for $\overline{\overline{\beta}} \leq 1$ the following condition must hold: $\frac{\omega^2-z}{(1-z)\omega} \leq \alpha \leq \omega$.

This implies that $\forall H_{1t} \geq \overline{\overline{H}}$ the low-growth equilibrium does not exist for all $\varepsilon \leq \theta \leq \omega$.

We now compare the two thresholds of the inherited human capital found above in a) and b) and show that $\overline{\overline{H}} \geq \overline{H}$.

$$\overline{\overline{H}} \geq \overline{H}$$

$$\left(\frac{\ln\left(\frac{1-\alpha\omega}{1-\omega^2}\right)}{\beta_0 \ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]} \right)^{\frac{1}{\lambda}} \geq \left(\frac{\ln\left(\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right)}{\beta_0 \ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} \right)^{\frac{1}{\lambda}}$$

$$\alpha \leq \frac{\omega(2\omega-1)}{1-\omega(1-\omega)}$$

Given that $0 \leq \alpha \leq 1$ and $\omega \rightarrow 1$, the condition above will always hold. Hence $\forall H_{1t} \geq \overline{\overline{H}} \implies \overline{\overline{\beta}} \leq \beta_t \leq 1$, there is a unique high-growth equilibrium given that $\alpha \geq \overline{\alpha} = \frac{\omega^2 - z}{(1-z)\omega}$.

(ii) Second, we show that when the level of inheried human capital $0 \leq H_{1t} \leq \underline{\underline{H}} \Longrightarrow \widetilde{\beta} \leq \beta_t \leq \underline{\underline{\beta}}$, then there is a unique low-growth equilibrium.

a) Using (C-3), by substituting in for β_t from (15), and by solving for H_{1t} we get

$$H_{1t} \leq \left(\frac{\ln\left(\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right)}{\beta_0 \ln\left[\frac{1-\varepsilon^2}{(1-z)[1-\alpha\varepsilon]}\right]} \right)^{\frac{1}{\lambda}} \equiv \underline{H}$$

where the threshold $\underline{H}$ of inherited human capital corresponds to the threshold $\underline{\beta}$ of patience in (C-3). Based on our results in Appendix C, this implies that $\forall H_{1t} \leq \underline{H}$ the low-growth equilibrium exists for all $\varepsilon \leq \theta \leq \omega$.

b) Using (C-4), by substituting in for β_t from (15), and by solving for H_{1t} we get

$$H_{1t} \leq \left(\frac{\ln\left(\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right)}{\beta_0 \ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} \right)^{\frac{1}{\lambda}} \equiv \underline{\underline{H}}$$

where the threshold $\underline{\underline{H}}$ of inherited human capital corresponds to the threshold $\underline{\underline{\beta}}$ of patience in (C-4). Based on our results in Appendix C, this implies that $\forall H_{1t} \leq \underline{\underline{H}}$ the high-growth equilibrium does not exist for all $\varepsilon \leq \theta \leq \omega$.

We now compare the two thresholds of the inherited human capital found above in a) and b) and show that $\underline{\underline{H}} \leq \underline{H}$.

$$\underline{\underline{H}} \leq \underline{H}$$

$$\left(\frac{\ln\left(\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right)}{\beta_0 \ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} \right)^{\frac{1}{\lambda}} \leq \left(\frac{\ln\left(\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right)}{\beta_0 \ln\left[\frac{1-\varepsilon^2}{(1-z)[1-\alpha\varepsilon]}\right]} \right)^{\frac{1}{\lambda}}$$

$$\alpha \geq \frac{-\varepsilon(1-2\varepsilon)}{1-\varepsilon(1-\varepsilon)}$$

Given that $0 \leq \alpha \leq 1$ and $\varepsilon \rightarrow 0$, the condition above will always hold. Hence, $\forall 0 \leq H_{1t} \leq \underline{\underline{H}} \Longrightarrow$

$\widetilde{\beta} \leq \beta_t \leq \underline{\underline{\beta}}$, there is a unique low-growth equilibrium.

(iii) Third, we show that when the level of inherited human capital $\overline{\overline{H}} > H_{1t} > \underline{\underline{H}}$, then there are multiple equilibria. In this case, there are three sub-cases, a) $\overline{\overline{H}} > H_{1t} > \overline{H}$, b) $\underline{H} > H_{1t} > \underline{\underline{H}}$, and c) $\overline{H} > H_{1t} > \underline{H}$.

a) $\forall \overline{\overline{H}} > H_{1t} > \overline{H} \implies \overline{\overline{\beta}} > \beta_t > \overline{\beta}$, $\overline{\theta} > \omega$, and $\varepsilon \leq \underline{\theta} < \omega$ (this also requires $\overline{H} \geq \underline{H}$). There will be a unique high-growth equilibrium $\forall \theta < \underline{\theta}$, and for $\underline{\theta} \leq \theta \leq \omega$ both the high-growth and the low-growth equilibrium coexist.

[Figure 1 here]

We also show that $\overline{H} \geq \underline{H}$

$$\overline{H} \geq \underline{H}$$

$$\left(\frac{\ln\left(\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right)}{\beta_0 \ln\left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} \right)^{\frac{1}{\lambda}} \geq \left(\frac{\ln\left(\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right)}{\beta_0 \ln\left[\frac{1-\varepsilon^2}{(1-z)[1-\alpha\varepsilon]}\right]} \right)^{\frac{1}{\lambda}}$$

$$\alpha \leq \frac{2\omega^2(1-\varepsilon^2) - \varepsilon^2(1+\omega)}{\omega^2(1-\varepsilon^2) - \varepsilon(1+\omega)}$$

Since $\varepsilon < 1$ and $0 \leq \alpha \leq 1$, this always holds.

b) $\forall \underline{H} > H_{1t} > \underline{\underline{H}} \implies \underline{\beta} > \beta_t > \underline{\underline{\beta}}$, $\underline{\theta} \leq \varepsilon$, and $\varepsilon \leq \overline{\theta} < \omega$. There will be a unique low-growth equilibrium $\forall \theta > \overline{\theta}$, and for $\varepsilon \leq \theta \leq \overline{\theta}$ both the high-growth and the low-growth equilibrium coexist.

[Figure 2 here]

c) $\forall \overline{H} > H_{1t} > \underline{H} \implies \overline{\beta} > \beta_t > \underline{\beta}$, $\varepsilon \leq \overline{\theta} < \omega$, and $\varepsilon \leq \underline{\theta} < \omega$. There will be a unique high-growth equilibrium $\forall \theta < \underline{\theta}$, a unique low-growth equilibrium $\forall \theta > \overline{\theta}$ and for $\underline{\theta} \leq \theta \leq \overline{\theta}$ both the high-growth and the low-growth equilibrium coexist.

[Figure 3 here]

(iv) Partial derivatives of the inherited human capital thresholds with respect to z and α

Partial derivatives of $\underline{\underline{H}}$ with respect to z and α.

$$\frac{\partial \underline{\underline{H}}}{\partial z} = \frac{-\underline{\underline{H}}^{\frac{1}{\lambda}}}{\lambda(1-z)\ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} < 0$$

$$\frac{\partial \underline{\underline{H}}}{\partial \alpha} = \frac{-\varepsilon^2 \underline{\underline{H}}^{\frac{1}{\lambda}}}{\lambda\left[1+\varepsilon+(2-\alpha)\varepsilon^2\right]} \left[\frac{1}{\ln\left[\frac{1+\varepsilon}{(1-z)[1+\varepsilon+(2-\alpha)\varepsilon^2]}\right]} + \frac{1}{\ln\left[\frac{1+\varepsilon+(2-\alpha)\varepsilon^2}{1+\varepsilon}\right]} \right] < 0$$

Taking partial derivatives of $\overline{\overline{H}}$ with respect to z and α.

$$\frac{\partial \overline{\overline{H}}}{\partial z} = \frac{-\overline{\overline{H}}^{\frac{1}{\lambda}}}{\lambda(1-z)\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]} < 0$$

$$\frac{\partial \overline{\overline{H}}}{\partial \alpha} = \frac{-\omega \overline{\overline{H}}^{\frac{1}{\lambda}}}{\lambda(1-\alpha\omega)} \left[\frac{1}{\ln\left[\frac{1-\omega^2}{(1-z)(1-\alpha\omega)}\right]} + \frac{1}{\ln\left[\frac{1-\alpha\omega}{1-\omega^2}\right]} \right] < 0$$

Taking partial derivatives of $\underline{H}$ with respect to z and α.

$$\frac{\partial \underline{H}}{\partial z} = \frac{-\underline{H}^{\frac{1}{\lambda}}}{\lambda (1-z) \ln \left[\frac{1-\varepsilon^2}{(1-z)(1-\alpha\varepsilon)}\right]} < 0$$

$$\frac{\partial \underline{H}}{\partial \alpha} = \frac{-\varepsilon \underline{H}^{\frac{1}{\lambda}}}{\lambda (1-\alpha\varepsilon)} \left[\frac{1}{\ln \left[\frac{1-\varepsilon^2}{(1-z)(1-\alpha\varepsilon)}\right]} + \frac{1}{\ln \left[\frac{1-\alpha\varepsilon}{1-\varepsilon^2}\right]}\right] < 0$$

Taking partial derivatives of $\overline{H}$ with respect to z and α.

$$\frac{\partial \overline{H}}{\partial z} = \frac{-\overline{H}^{\frac{1}{\lambda}}}{\lambda (1-z) \ln \left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} < 0$$

$$\frac{\partial \overline{H}}{\partial \alpha} = \frac{-\omega^2 \overline{H}^{\frac{1}{\lambda}}}{\lambda [1+\omega+(2-\alpha)\omega^2]} \left[\frac{1}{\ln \left[\frac{1+\omega}{(1-z)[1+\omega+(2-\alpha)\omega^2]}\right]} + \frac{1}{\ln \left[\frac{1+\omega+(2-\alpha)\omega^2}{1+\omega}\right]}\right] < 0$$

Appendix E

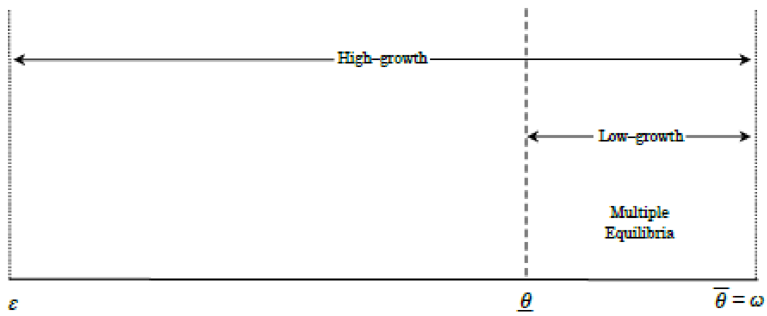

Figure 1

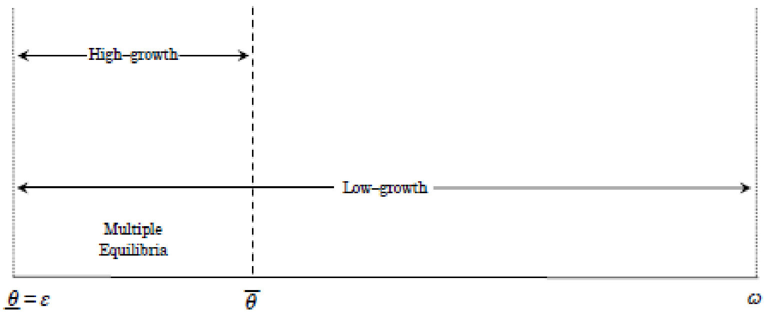

Figure 2

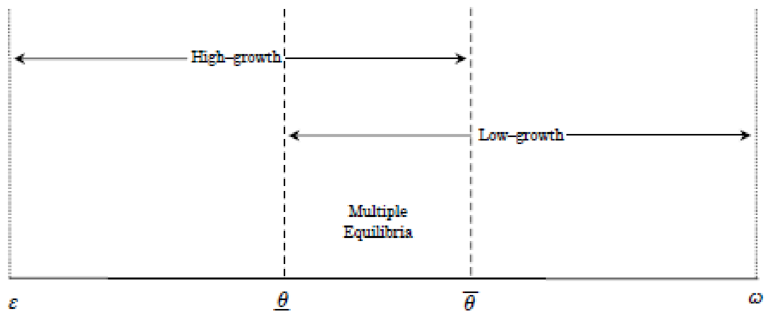

Figure 3